

Fig. 6. A model of the first Brillouin zone of a body-centered cubic lattice. The sphere occupies half the volume of the zone.

situation is illustrated in Fig. 6, which shows a model of the first Brillouin zone of a body-centered cubic lattice containing a sphere whose volume is just half that of the zone. Figure 7 shows a two-dimensional square lattice, its two-dimensional Brillouin zone and the Fermi "circle" having an area of just half that of the zone. In Fig. 7 (a, b, c, and d) I have shown, purely schematically, the progressive distortion of the Fermi surface; in d this surface is in marked contact with the Brillouin zone. In general, distortion of the Fermi surface causes those regions which are nearest the zone boundaries to become even closer.

A great deal is now known about the Fermi surfaces of the noble metals from

a variety of techniques which give direct information about the shape and other features of the Fermi surface (16). These methods agree in showing that the Fermi surfaces in copper, silver, and gold all touch the zone boundary. About the alkali metals we have as yet no direct evidence, but indirect evidence suggests that the Fermi surfaces of sodium and potassium are nearly spherical, that the Fermi surface of rubidium is somewhat distorted, and that the Fermi surfaces of lithium and cesium are much more distorted, perhaps touching the Brillouin zone boundary (17, 18). The effects of such distortion on electrical resistivity are discussed later.

When an electric field is applied to a metal the conduction electrons are accelerated and the whole Fermi surface begins to move in the direction of the field (see Fig. 8). The electrons, however, are prevented from continuous acceleration in the field by collisions with phonons (we are considering only the ideal resistivity), and the Fermisurface movement is almost vanishingly small. The effect of the distortion of the Fermi surface on the scattering of electrons by phonons is a difficult theoretical problem, and detailed studies have only recently been made (19). One of the most important effects arises from a type of scattering process called an "Umklapp" process, which gives rise to large angle scattering of the electrons.

First consider a typical normal scattering process in which an electron of



'/d

Fig. 7. A two-dimensional square lattice and the corresponding first Brillouin zone. (a) The Fermi "circle" corresponding to one electron per atom; (b), (c), and (d), progressive distortions of the Fermi surface (schematic).

wave vector k is scattered by a phone of wave vector q into a new state wave vector k'; k, k', and q are relate by the vector condition that k - k' = k'Moreover, the phonon energy at a normal and low temperatures is vesmall compared with the Fermi energ of the electrons. Since only those ele trons near the Fermi level have neigh boring unoccupied states into which they may be scattered, the scattere electron must both start and end effe, tively at the Fermi surface. Figure shows the geometry of a normal scatte ing process. As mentioned earlier, th Brillouin zone governs the behavior all kinds of waves that can propaga through the metal, including lattic waves; the biggest wave vector that phonon can have is one which reach from the center of the zone to the zone boundary. This therefore lim the angle through which an electro may be scattered in a normal proces even at the highest temperatures. A low temperatures, where only low energy phonons (having therefore sma wave vectors) are excited, the angle scattering is even further limited in suc processes.

An Umklapp process may be interpreted as one in which the electron scattered by a phonon and also under goes a Bragg reflection. In vector term the well-known Bragg condition is represented by the equation k' - k = i where R is a reciprocal lattice vector In Fig. 7 the vectors R' and R'' are two reciprocal lattice vectors for the simple square lattice. Thus, in an Umklap process the vector condition

$$k' - k \equiv q$$

is replaced by

$$k'-k \equiv q+R$$

where R is a reciprocal lattice vecto Such a process is illustrated in Fig. 1 Its importance lies in the fact the because the large vector R enters in the process, it makes possible scattering at wider angles than can occur in normal process. This can also be set by a geometrical construction. Eqution 8 may be rewritten as

$$k + R = k' - q$$

and we begin by representing graphically all the possible vectors k+i. Since the k vectors of all electron which can be scattered must lie on the Fermi surface, the vectors k + R', is example, must lie on the same surface displaced by the vector R'; the same

SCIENCE, VOL. 14 JULY

Fermi Fermi illustr: Umkla If th the zo a certa klapp minim of clo Fermi tor CL ess wi the el angle which usually ing ar process compa The q for 1 low te proces: the nu enough Thus, can in ence o

Peratur

of the

tures th

approa

true

vector

tors li

of the

displac

satisfy

of ene

trons

origina

Eq. 8.

which

82